INVESTIGATION OF TAPERED MULTIPLE MICROSTRIP LINES FOR VLSI CIRCUITS

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ABSTRACT

The S-parameters of coupled, tapered microstrip etches are calculated as a function of frequency using an iteration-perturbation technique. The propagation constants and impedance matrices are computed using a perturbation-iteration approach. Representative results are obtained to illustrate the application of the method.

INTRODUCTION

Tapered microstrip lines are commonly used in VLSI circuits for matching lines of different width and in routing of lines from chip packages to board-level interconnects.

The tapered transmission line has been studied by Hill and Mathews (1) and Rao, et al (2). In their investigations, a quasi-TEM approach was adopted. An accurate full-wave analysis of tapered lines has been proposed by Mirshekar-Syahkal and Davies (3) using the the method of coupled modes accompanied by the spectral domain solution of uniform lines. However, finding the eigenvalues of the normal modes for the uniform line is time consuming and not convenient for computer-aided design. Recently, Kretch and Collin (4) developed a numerically efficient iteration-perturbation theory to study the dispersion effect of the dominant propagating mode of a microstrip line.

In this presentation, we calculate the S-parameters of a tapered microstrip line as a function of frequency using the approach described in (4) for the uniform lines. Although the coupled-mode method (3) could be used if the higher-order modes were excluded, the impedance technique (5) is adopted for programming simplicity.

PROCEDURE

The tapered microstrip line is analyzed by dividing the line into small segments so that each segment can be approximated as a uniform line. For each frequency, starting with the static case, the effective dielectric constant, $\varepsilon_{r,eff}$, is determined. This value is then used to compute the characteristic impedance of that section of line and the S-parameters are obtained using standard microwave analysis. Next, the S-matrix is converted into a T-matrix, and since the sections are cascaded, all T-matrices are multiplied to derive a final T-matrix. This final T-matrix is then converted into an S-matrix, which is frequency dependent, and the mismatch introduced by the taper is obtained from the S-matrix.

The frequency-dependent effective dielectric constant $\varepsilon_{r,eff}$ is determined using an iterative approach described by Kretch and Collin (4). We begin by computing the static $\varepsilon_{r,eff}$ and use it to obtain the potentials on the microstrip. Next, a new $\varepsilon_{r,eff}$ is found for these potentials and the procedure is repeated until the value of $\varepsilon_{r,eff}$ converges. This value is used to find the characteristic impedance, and the initial estimate for $\varepsilon_{r,eff}$ at the next frequency increment.

THEORY

Transmission Line Theory

The tapered microstrip line is approximated by dividing it into uniform sections. For each section, the S parameters are calculated as defined by microwave network theory.

$$S_{11} = -S_{22} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{1}$$

$$S_{12} = S_{21} = \frac{2\sqrt{Z_1 Z_2}}{Z_2 + Z_1} \tag{2}$$

After the S parameters for the discontinuity are calculated, the terminal plane is translated through a distance equal to the electrical length of the uniform section. This amounts to multiplying each term by a phase shift of $e^{j\beta l}$ (5). After the phase-shifted parameters are found, the S-matrix is transformed into a T-matrix as follows:

$$T_{11} = \frac{S_{11}\, S_{21} - S_{11}\, S_{22}}{S_{21}} \tag{3}$$

$$T_{12} = \frac{S_{11}}{S_{21}} \tag{4}$$

$$T_{21} = \frac{-S_{22}}{S_{21}} \tag{5}$$

$$T_{22} = \frac{1}{S_{21}}. \tag{6}$$

Next, a composite T-matrix is derived by forming the product of all T-matrices to this point. This procedure is repeated until all sections have been included. The final T-matrix is then converted back into an S-matrix. For the three line geometry, we treat the system as a six-port microwave network, in which case equations (1) - (4) become matrix equations.

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Frequency dependence enters into the elements of the S-matrix in two ways. First, because the propagation constant β varies with frequency, the electrical length by which the terminal plane must be shifted differs for each frequency. This can lead to different S-parameters for different frequencies. Second, Z_0 and ε_r are functions of frequency and contribute to the frequency dependence of the S-matrix.

Potential Theory

The $\varepsilon_{r,eff}$ and Z_0 for the single line is computed using the procedure in (4). The effective dielectric constant for the three-line geometry shown in Figure 1 can be found by using the same potential theory. The current distribution $J_z(x)$ and charge distribution $\rho(x)$ for each line are expanded into basis functions, and the Green's function is determined for the given geometry. We start by assuming a uniform potential on each strip, which in general consists of three different potential distributions corresponding to three different modes: two even and one odd. $J_z(x)$ and $\rho(x)$ must be expanded in terms of suitable basis functions for each mode. The Green's function, however, remains the same for all three modes. But it can be separated into an even and an odd part, of which only one part is needed for each mode. The integral equations for the potentials are then derived for each mode and are converted into matrix equations using the method of moments. The matrix equation is then solved for the coefficients of the basis functions. These coefficients are used to determine the current and charge densities, which enables us to find the total current I_T and total charge Q_T on each conductor. We then find the effective dielectric constant $\varepsilon_{r,eff}$ for each mode by finding the eigenvalues of the matrix $\mathbf{QI^{-1}}$.

Using this $\varepsilon_{r,eff}$, we can find a new estimate of the potentials on the strips, which allows us to add a perturbation term to the right hand side of the integral equation to account for the current in the x direction. We then solve the equations again using the new estimate of the potential distribution. The procuedure is repeated until successive values of $\varepsilon_{r,eff}$ do not change by more than 0.1%, at which point we step to the next frequency.

RESULTS

Single Line

The procedure described above was tested by finding the reflection coefficient for the single-line taper shown in Figure 2. The nominal value of ε_r varied from 1.0 to 12.9, and the results were obtained for both the frequency dependent $\varepsilon_{r,eff}$ and $\varepsilon_{r,eff}$ derived under the static approximation (marked by an x on the graph). In all cases, the taper was linear with a length of 1 cm., but the procedure can be used with any arbitrary shape and length. The results are shown in Figures 3 through 8. Note that the reflection coefficient is plotted on a log scale, and the frequency dependence affects the position of the minimums more than the magnitude of the relflection coefficient.

Three Lines

The propagation constants for a uniform three line structure was determined. A comparison with the propagation constants obtained by spectral Galerkin procedure (6) are shown in Table 1. The values differ by less than 0.5%, and this procedure required less computation time. The quasi-static current distributions for each mode are shown in Figures 9 through 12.

For the three tapered microstrip lines, the S-matrix for the coupled lines becomes a six-port network. The major difference from the one-line case comes from the way the effective dielectric constant is evluated for the multiline system. The potential theory described above accounts for the neighboring lines and for the possibility of even and odd modes.

CONCLUSIONS

A perturbation-iteration method for determining the S-parameters of three tapered microstrip lines based on potential theory has been presented. Representative results are obtained to illustrate the application of the method for a single tapered line. This method appears to be well-suited for efficient evaluation of the mismatch effect of tapered etches on digital pulses.

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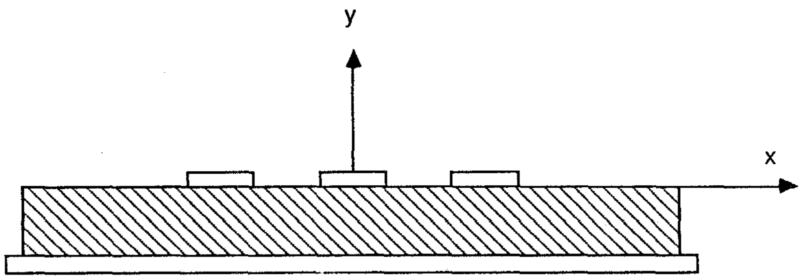

Figure 1. Cross section of three-line microstrip

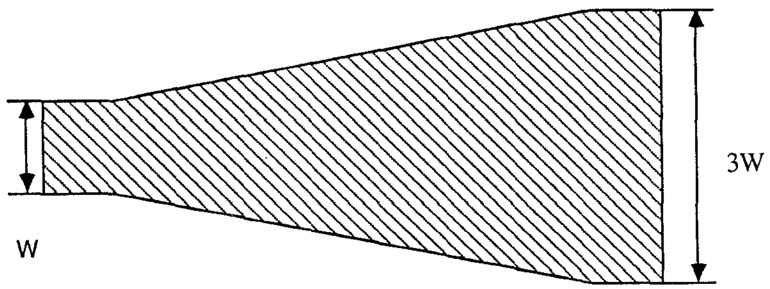

Figure 2. Tapered Microstrip Line

TABLE 1

Comparison of Propagation Constants

	This Method	From (6)
$\beta_{e,1}$	50.71	50.53
$\beta_{e,2}$	57.92	58.12
β_o	52.98	52.80

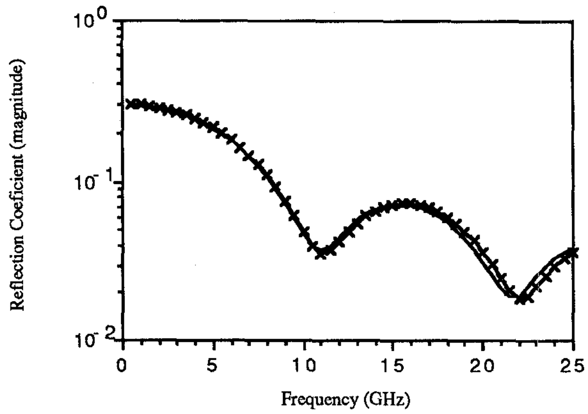

Figure 4. Linear taper, $\varepsilon_r = 2.2$, length = 1 cm.

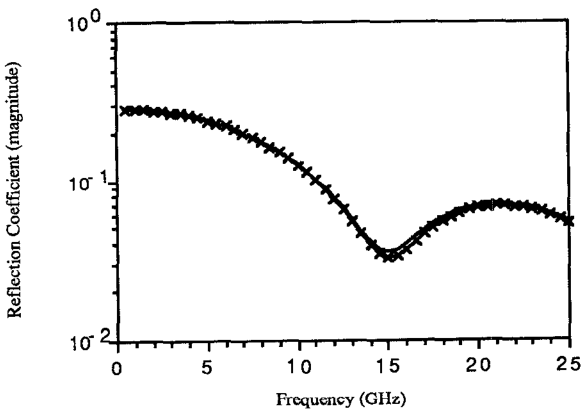

Figure 3. Linear taper, $\varepsilon_r = 1.0$, length = 1 cm.

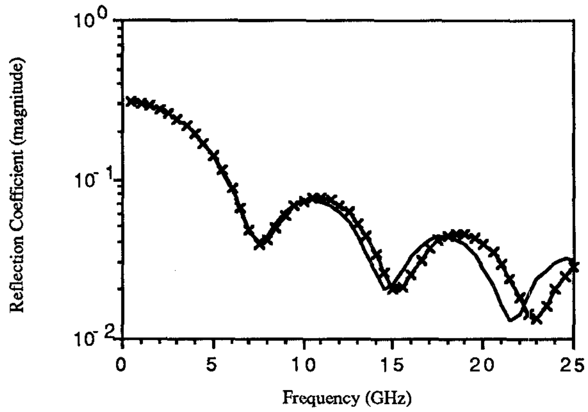

Figure 5. Linear taper, $\varepsilon_r = 5.12$, length = 1 cm.

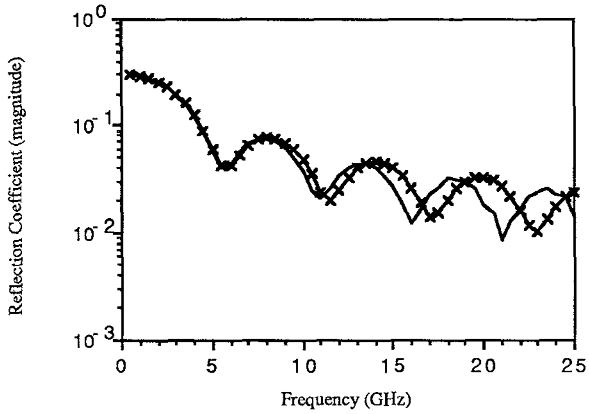

Figure 6. Linear taper, $\varepsilon_r = 9.4$, length = 1 cm.

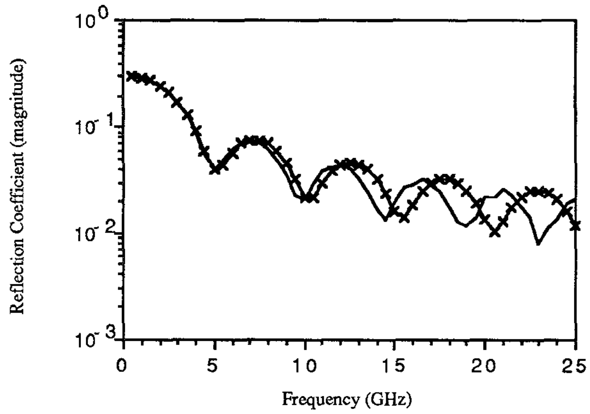

Figure 7. Linear taper, $\varepsilon_r = 11.8$, length = 1 cm.

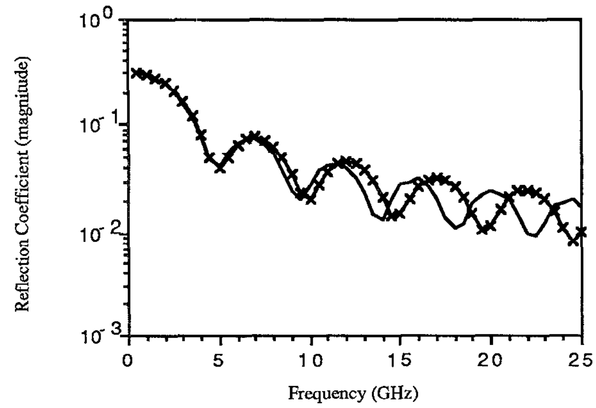

Figure 8. Linear taper, $\varepsilon_r = 12.9$, length = 1 cm.

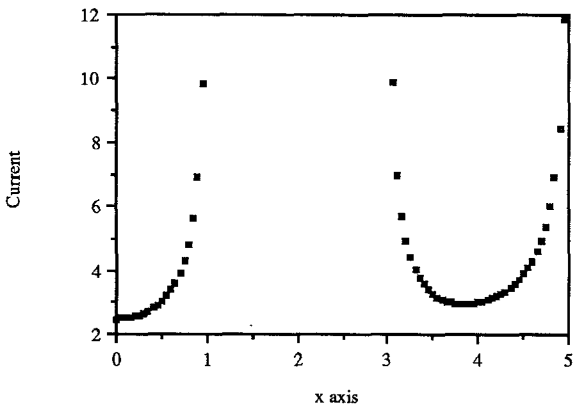

Figure 9. Current distribution for (1,1,1) mode

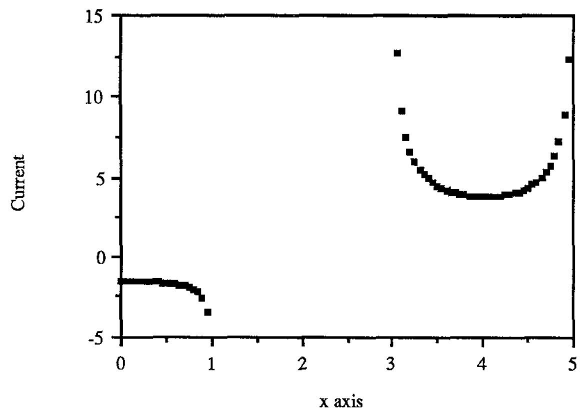

Figure 10. Current distribution for (1,0,1) mode

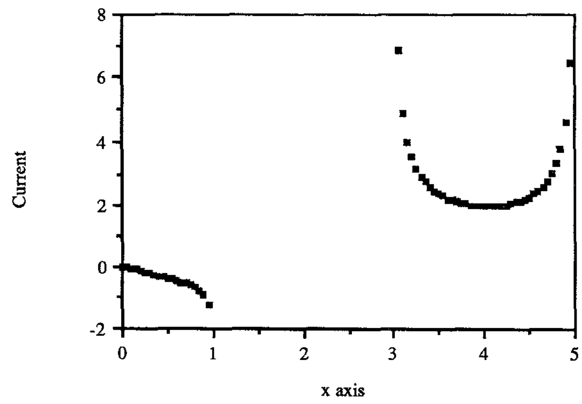

Figure 11. Current distribution for (-1,0,1) mode